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Regional-scale Spatio-Temporal Analysis of Impacts of Weather on Traffic Speed in Chicago using Probe Data

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Abstract

Understanding a regional-scale impact of the weather on the transportation system and how that impact varies geographically, is important from a sustainability standpoint. In this work, we have performed a city-scale analysis on the impact of weather on the traffic speed for the City of Chicago. We have found that there is a significant variation in the average hourly speed due to different weather patterns which also varies geographically. We have also observed that the rainfall has an evident impact on the average hourly speed on the freeways that are influenced by urbanized residential and commercial areas, and non-urbanized areas. Also, low visibility has shown a significant reduction in the average hourly traffic speed during the congested hours on the freeways that are influenced by non urbanized areas. We anticipate the contributions of this work in estimating emission and fuel economy at a regional scale which are important sustainability measures for transportation.

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1. Introduction

Sustainability of the transportation system has been a growing concern among both researchers and policy makers, which mainly deals with environmental, economical and social sustainability [1]. Environmental sustainability addresses the environmental impacts of the transportation system whereas economical sustainability concerns about the fuel consumption and cost of person time spent in the transportation. The social sustainability deals with the transportation accessibility and mobility. Bigazzi and Bertini have developed performance metrics such as emission, energy consumption, delay cost and person mobility, to measure the sustainability of the transportation system [1]. According to the EPA report, transportation sector accounts for the largest fraction (28%) of total US green house gas (GHG) emission in year 2016 [2, 3] and also for 28.5% of total energy consumption in the US [4]. According to the federal highway administration (FHWA) report, the total cost of delay in transportation in the year 2012 was \$154 Billion [5].

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Weather is one among the many factors that impacts the above mentioned sustainability performance metrics. For example, during inclement weather drivers tend to drive slowly which reduces the freeflow speed and increases the traffic delay. Although not listed above, the traffic safety is also an important sustainability measure. As per the FHWA, 24% of the total crashes are caused by the adverse weather conditions. Due to this close relationship of weather with transportation system, there have been active efforts from both government as well as research community to study the impacts of the weather events on the transportation system. A study was carried out to analyze the impacts of weather events on traffic operations, to identify the key weather related parameters in a traffic simulation model CORSIM and to perform sensitivity analysis of these parameters [6]. Bartlett et al. have studied the impact of inclement weather on hourly traffic volumes and developed a mathematical model to predict the traffic volumes in the Buffalo, New York region based on inclement weather forecast [7]. A similar work showed that there is a negative correlation in the range of -0.25 to -0.32 between snowfall amount and the daily traffic volume in the western New York region [8]. Recently, Zang and Chen have used a non-parametric data mining technique called classification and regression tree (CART) to discover various patterns in the travel times using data of good weather days. Further, various weather patterns in each time group were obtained and a regression analysis was done to analyze the impact of rainfall and snow on travel time in different time periods [9]. In a similar study, Zhang et al. have analyzed the impact of the rainfall on the traffic speed in Beijing, and developed an exponential model for freeflow speed under rainfall conditions [10].

As discussed above, several studies have been reported in the past that analyze the impacts of the inclement weather on traffic flow characteristics, freeflow speed and capacity. We have noticed that, most of these studies were carried out for a specific corridor or for a freeway segment of a specific length. There is a paucity of the regional-scale spatial-temporal studies on the impacts of the inclement weather on the transportation system. It is important to understand, how the impacts of different weather patterns on traffic flow varies geographically and consequently, its implications on sustainability measures such as emission and energy consumption. As mentioned by Zang and Chen [9], from a commuter's standpoint, having an information on the excessive delay due to congestion on their way to work is very important. However, traffic congestion varies spatially as well as temporally. For instance, during a trip to the work, a commuter may take different road facility types (freeways, arterial and collectors) and also drive through various neighborhoods having different land-use types such as urbanized, non-urbanized and residential areas. The commuter trips either generate from or terminate to these different land-use categories. For example, a commuter may start the trip from home (possibly, a residential area) and terminates it to the work place (possibly, a non-residential area). This explains, how different land-use types have localized effect on the traffic flow and congestion on the nearby road segments. Furthermore, the traffic flow characteristics vary throughout the day. Due to these reasons, these factors, i.e. geography and time, should both be taken into account while performing a regional-scale impact analysis of the weather on a transportation system.

In this work, we have performed a city-scale analysis on the impact of weather on the traffic speed for the City of Chicago. We have found that, there is a significant variation in the impact of different weather patterns on the average hourly traffic speed (AHTS) and it varies geographically. This paper is organized as follows: [section 2](#) describes the different datasets used in the study. In [section 3](#), we present our methodology followed by, in [section 4](#), a discussion on the various results we obtained in the analysis. Finally, [section 5](#) concludes the paper by describing major highlights of the work and provides future directions.

2. Study area and data used

This study was performed for the City of Chicago for the year 2017. Chicago has a typical continental weather with cold winter and warm summer with frequent short fluctuations in weather. The dataset used for this study mainly consists of land-use data, weather data and vehicle probe data for the Chicago region. The land use data used in this work is provided by Chicago Metropolitan Agency for Planning (CMAP) [11]. CMAP periodically conducts a survey of the region's land-use and publishes the land-use inventory, which CMAP uses for the land-use and transportation research. For this study, we have used the land-use inventory that was published in the year 2013. We have used vehicle's speed and travel time data from the dataset called as National Performance Management Research Data Set (NPMRDS [12]) which is procured and sponsored by Federal Highway Administration (FHWA). The NPMRDS data mainly contains information about average vehicle speed, link length, free-flow speed. We have downloaded hourly data for the year 2017. The data is available for 1589 links covering entire city and consist of different facility types.

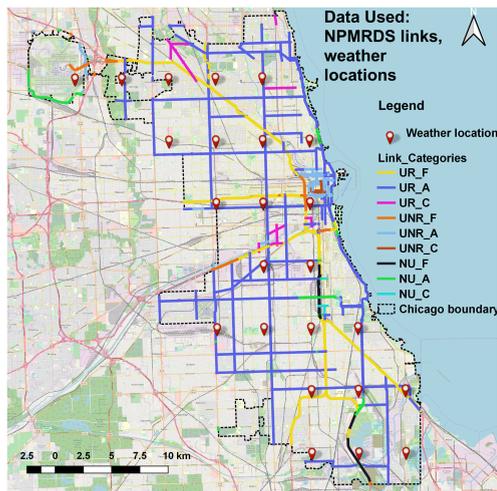
Table 1: Shows the summary statistics of the data used in this work

		percentage	mean	s.d.	min	max	count
Weather data	Temperature (°F)	–	52.18	19.46	-8.70	94.18	–
	Precipitation Intensity (inch)	–	0.003705041	0.0194	0.00	0.565	–
	Visibility (miles)	–	9.27	1.94	0.24	18.92	–
	Wind speed (mph)	–	4.29	2.68	0.00	10.00	–
NPMRDS speed data	Freeway (mph)	–	48.12	15.51	3.00	99.00	424
	Arterial (mph)	–	21.07	11.86	3.00	99.00	1101
	Collector (mph)	–	14.64	9.14	3.00	98.00	64
Land-use data	Urban residential	30%	–	–	–	–	–
	Urban Non-residential	33%	–	–	–	–	–
	Non Urban	30%	–	–	–	–	–

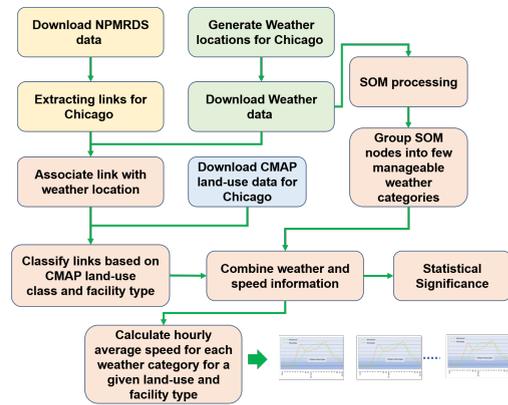
Figure 1 (a) provides the overview of the datasets used in this study. To account for the local variation in the city’s climate, we have selected 24 grid point locations which cover entire City of Chicago and these points are separated by 0.05°. Figure 1 (a) shows these weather locations. The hourly weather data was downloaded for all of these 24 locations for the year 2017 through Dark Sky Application Programming Interface (API) [13]. The major weather attributes that are considered in this work are precipitation intensity, visibility, wind speed and temperature. Table 1 presents few statistics about these datasets.

3. Methodology

In this work, we have preformed a city-scale analysis of the variations in the traffic speed due to different weather patterns across the city of Chicago. The land-use type is one of the important factors that impacts the traffic flow of the nearby road network. Moreover, due to the geography, the city of Chicago experiences micro-climates, which also has a localized impact on the traffic flow. Therefore, we have considered both weather and land-use factor in our analysis. Figure 1 (b) shows various steps involved in this work, which are described below in this section.



(a)



(b)

Fig. 1: (a) Datasets used in this work. The weather locations (0.05° grid), represents spatial locations for which the weather data was downloaded, are also shown. The NPMRDS links for which speed data was available for the city of Chicago are shown. These links are coloured based on the most influencing land-use type (UR:“Urbanized Residential”, UNR:“Urbanized Non-Residential”, NU:“Non-Urbanized”) and the link’s facility type (F:“Freeway”, A:“Arterial”, C:“Collector”). There are total nine categories in which the road links are classified. CMAP’s land-use classification scheme is used. (b) Shows various steps involved in the methodology of this study. The boxes in light orange color show various data processing steps and the boxes in other colors represent various data acquisition steps where different type of data are represented in different color (other than light orange).

3.1. Classification of the (NPMRDS) network links

To accommodate the impacts of the land-use type on the traffic flow, we have classified the network links based on the dominant land-use class in their vicinity and the facility type of the link. For this purpose, the land-use data obtained from CMAP and the facility type information of each link provided in the NPMRDS network were used. We have considered urbanized residential (UR), urbanized non-residential (UNR) and non-urbanized (NU) as land-use categories. The urbanized residential areas are predominantly used for the residential purpose whereas, urbanized non-residential lands are predominantly used for commercial, industrial or institutional activities. In contrast, non-urbanized land contains vacant parcels, under-construction lands, non-parcel areas. Along with the land-use classes, the facility types Freeways (F), Arterial (A) and Collector (C) were considered in our link classification scheme, due the difference in their speed limits and road characteristics. Collectively, we have defined nine categories for the classification of the links in the network. For instance, the link category “UR_F” represents a set of freeway links in the network that are mostly influenced by urbanized residential areas. The most influential land-use class for a given link in a network was identified by calculating the largest fraction of the land-use type within the 500 meter region around that network link. [Figure 1](#) (a) depicts the NPMRDS road links which are colored by the nine link categories.

3.2. Discovering weather patterns

As mentioned earlier, the main goal of this work is to study the impacts of the weather on the traffic speed in the city of Chicago. As a first task, we have discovered various weather patterns in Chicago. The weather data used for this task contains hourly weather information about temperature, precipitation intensity, visibility and wind velocity for year 2017 from 24 different locations across the city. These hourly weather patterns from all the 24 weather locations were combined and processed using an unsupervised classification technique called Self-Organizing Maps (SOM) [14]. SOM then mapped the higher-dimensional weather data into a lower-dimensional space, i.e. 2D grid of nodes, such that all the data points in a node shows similar hourly weather characteristics. A grid of size 10×10 nodes with hexagonal topology and a toroidal neighborhood structure was used. These SOM nodes were further grouped into few clusters such that all the hourly weather data points in a cluster represent similar weather pattern, e.g. good weather, high precipitation, low visibility, etc. The mean and standard deviation statistics of the weather attributes for different weather clusters were examined to characterize them according to the weather patterns. The generated SOM plot or clusters of weather patterns can be used to predict the weather pattern for any hour (in future) using the forecasted weather attributes for that hour.

3.3. Analyzing Speed variation due to weather

After the discovery of different weather patterns, it is important to study their impacts on traffic speed and also to analyze whether the difference in the average hourly speed due to different weather patterns are statistically significant. For this analysis, we have considered the speed data only for typical weekdays (i.e. Monday–Thursday) since maximum traffic speed variations can be observed during these weekdays and had distinct traffic flow characteristics than Fridays and weekends (Saturday–Sunday).

For a particular link category (say “UR_F”), we have carried out a statistical analysis test called ANOVA to check whether there is a significant variation in an average hourly traffic speed (AHTS) considering different weather patterns. If there is a significant variation in AHTS for a given link category, then more detailed test called tukey test was performed to determine which pairs of weather patterns are contributing to this overall variation. To perform this task, the hourly speed data of all the links of a specific category, say “UR_F”, was first combined in a single data object. An instance in this combined data object represents speed information of a particular link of category (“UR_F”) during a specific hour of some day in year 2017. Further, for each data instance, corresponding weather information was also appended in the combined data object. All the links in the data were already mapped to their nearest weather locations and this mapping was used in this task to combine the speed and corresponding weather information at an hourly resolution. Each hourly instance of the weather data was already classified and labeled according to a specific weather pattern. Finally, each data instance in this combined data object contains speed as well as weather pattern information during a specific hour of some day in year 2017. This combined data object was then used to carry out significance testing.

Algorithm 1 Procedure to obtain the average speed profiles for different weather patterns.

```

1:  $D_d^h \leftarrow (W_d^h, S_d^h)$ 
   Where,
    $D_d^h$ : the data instance on day  $d$  and at hour  $h$  of the day  $d$ 
    $W_d^h$ : weather data on day  $d$  and at hour  $h$  of the day  $d$ 
    $S_d^h$ : Speed information on day  $d$  and at hour  $h$  of the day  $d$ 
2:  $G \leftarrow$  Group data instances  $D$ ; based on corresponding weather instances (hourly)  $W_i$ 
3: for each group  $g \in G$  do
4:   for each hour  $H \in (1 : 24)$  do
5:      $gD_d^{h=H} \leftarrow$  Extract Hour  $H$  data instances  $\in$  group  $g$ 
6:      $avgSpeed^{h=H} \leftarrow$  Calculate average speed for hour  $H$  using data instances  $\in gD_d^{h=H}$ 
7:   end for
8: end for

```

The combined data objects of individual link categories were further used to generate the average diurnal speed profiles for weekdays (Monday–Thursday) for all weather patterns using Algorithm 1. For each weather pattern, the average speed is calculated for every hour of the day (refer line 3-7 in Algorithm 1). In this way, the obtained average diurnal speed profile for a specific weather pattern represents average (traffic) speed during an hour H of the day, which essentially captures the impact of weather pattern on traffic flow during that hour H .

4. Results and discussion

In this section, we present and discuss various results that were obtained during the analysis. First, we will show the weather patterns discovered through the SOM process and discuss the labels assigned to these weather patterns. Next, we present the results on significance testing on average hourly speed by considering all the weather patterns together for a given link category. Further, we show the diurnal speed profiles for all the weather patterns and link categories. Towards the end, we will discuss few implications on the emission and fuel economy.

4.1. Generated weather patterns and their labeling

Figure 2(a) shows the code plot (fan diagram) that was obtained after SOM processing of the weather data. Each SOM node in the plot represents the magnitude of each weather variable in that node. These SOM nodes were further grouped into five more general categories by using k -means clustering approach, which is shown in the Figure 2(b). Depending on the dominant weather attribute, each cluster was then assigned a label which represents a specific weather pattern (refer Figure 2). Figure 3 shows hourly mean and standard deviation of each weather attribute considered in this work for all the five different clusters. Cluster-1 has moderate temperature, no precipitation, low wind speed and high visibility. Therefore, cluster-1 was designated as “Good weather”. Cluster-2 has slightly below mod-

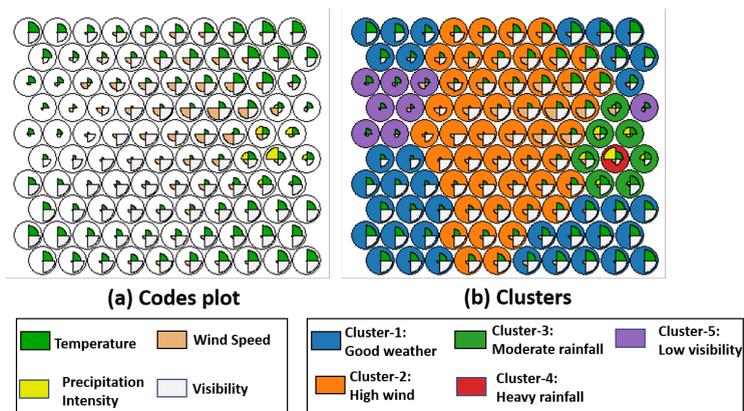


Fig. 2: (a) Shows Codes plot obtained after SOM processing. A fan plot in a node represents magnitude of the weather attributes. (b) Shows the results of the clustering applied over SOM grid. Total five clusters are obtained, each representing a unique weather pattern.

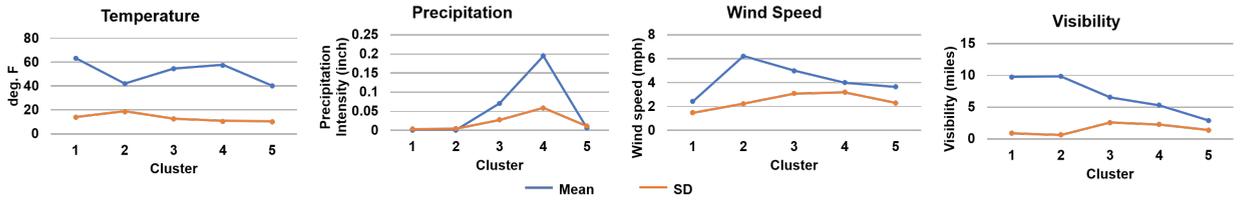


Fig. 3: Hourly Mean and SD plot of all the weather attributes for all the five clusters.

erate temperature, no precipitation, high wind velocity and good visibility. Therefore, cluster-2 was labeled as “High wind” cluster. We labeled cluster-3 as “Moderate rainfall” since cluster-3 shows moderate precipitation and moderate visibility, and moderate values for temperature and wind speed. Similarly, we labeled cluster-4 as “Heavy rainfall” as it shows high precipitation, slightly low visibility, moderate winds speed and slightly high temperature. Cluster-5 shows lowest visibility values among others, whereas, other three weather variable show moderate values. Therefore, we designate “Low visibility” label to cluster-5. From detailed inspection of the weather data, we have found that, the reason behind low visibility is fog and light shower (rain). The mean precipitation value for cluster-5 is 0.005380591 inch which also confirms the presence of the light precipitation.

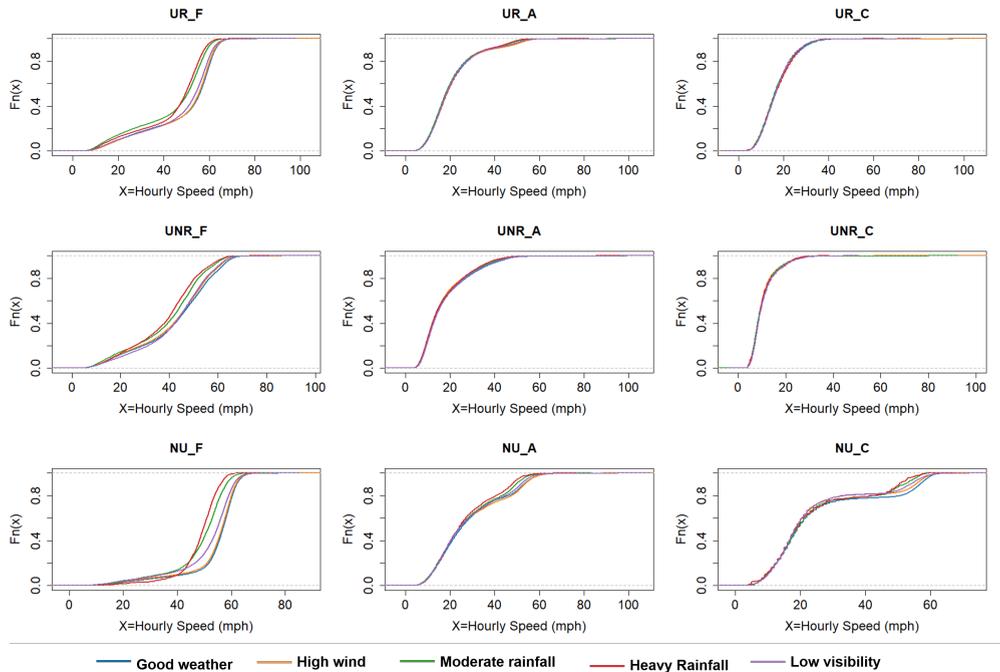


Fig. 4: Cumulative Density Functions of average hourly speed for all link categories. Weather patterns are represented by different line color.

4.2. Significance test results

As mentioned in the earlier section, we have carried out Anova test to check if there is a (statistically) significant difference in the average hourly traffic speed (AHTS) between different weather patterns that were obtained. This test was performed for all the link categories. For all the nine link categories, Anova showed p -value < 0.05 . Therefore, we can reject the null hypothesis with 95% confidence interval, which indicates that there is a significant variation in the AHTS between different weather patterns for all the link categories. We can make out similar observations from Figure 4 which presents CDFs of AHTS for different weather patterns for all link categories.

Anova test does not tell us which pairs of the weather patterns have significant variation. To identify this, we further carried out a post hoc test called Tukey test to perform pairwise comparison for AHTS for different weather patterns.

We observed that, the AHTS of freeway links those are influenced by urbanized residential area (i.e. “UR_F”), urbanized non-residential areas (i.e. “UNR_F”) and non-urbanized areas (i.e. “NU_F”) were significantly different (p-value < 0.05) for all the weather pattern pairs except for “Moderate rainfall” and “Heavy rainfall” (4_3) for link categories “UR_F” and “NU_F”. This can also be observed in the Figure 4. Most importantly, we can say that the variations of AHTS on freeways during weather events such as high wind, precipitation and low visibility is significantly different than that of good weather. Similar conclusions can be drawn about the variations of AHTS for arterial and collectors.

It should be noted that, we have used only the vehicle speed data (i.e. probe data), and weather is one out of the many other factors that could have impacted the average traffic speed. Through this analysis we have shown that, at the scale of the city, the weather has a significant impact on the average hourly traffic speed, which varies geographically.

4.3. Average weekday’s diurnal speed profiles

We have generated the average diurnal speed profiles by using Algorithm 1 for all the link categories. Due to the limitation of the space, we have presented the speed profiles only for freeways (refer Figure 5). We have combined cluster3 and 4, i.e. moderate and heavy rainfall cluster’s speed data due to the fact that they did not show significant difference in the significant test. We designate the combined cluster as 3+4 in the figure.

It can be observed that, the speed profiles of freeways that are influenced by urbanized residential area, i.e. “UR_F” show two dips corresponding to the peak hours at around 7:30AM and 5:30PM. Whereas, the speed profiles of links that are influenced by the urbanized non-residential areas such as commercial and industrial areas (“UNR_F”) and non-urbanized areas such as vacant/under construction areas (“NU_F”) does not show evident dip in the average speed in the morning time. The diurnal speed profiles corresponding to the cluster-3+4, i.e. rainfall, shows noticeable reduction in the traffic speed. Also, the diurnal speed profile of freeway influenced by non-urbanized areas shows reduction in the traffic speed due to low visibility.

These diurnal speed profiles can be used in the traffic simulation models to simulate the impacts of weather (good or inclement) on traffic flow. In future, we intend to use forecasted weather information to generate the speed profiles for (future) scenarios for simulation models.

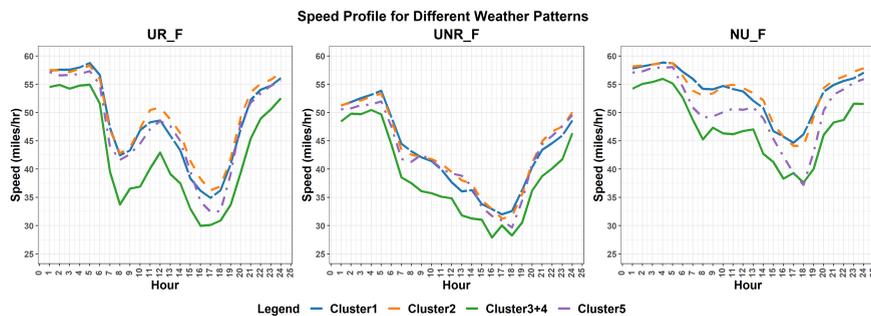


Fig. 5: Diurnal speed profiles showing impacts of the different weather on average traffic speed. “Cluster1” represents Good weather, “Cluster2” represents High Wind, “Cluster3+4” represents Rainfall and “Cluster5” represents low visibility due to the fog and light showers.

4.4. Emission and fuel economy

The past studies showed that the vehicle emissions and the fuel consumption that are causing those emissions depend on several factors such as driving behavior, roadway types and traffic conditions. The average speed is one of the important factors that determines the traffic performance. There is a relation between emission and average traffic speed. An average traffic speed less than the freeflow speed causes congestion, which means vehicle spends more time on the road and that leads to more emission per mile and more fuel consumption. The analysis carried out in this work can give us an approximate estimate of emission and fuel consumption for different weather patterns.

According to [3], if congestion on a freeway reduces the average vehicle speed below 45 mph, CO₂ emission increases, which also indicates the increased fuel consumption. The diurnal profiles in Figure 5 show that, during the congestion hours, the average weekday’s traffic speed reduces below 45 mph, and further reduces down to 30 mph due to rainfall for freeways that are influenced by the urban residential areas and urban non-residential areas.

For the freeways that are influenced by the non-urbanized areas the speed profiles shows that the low visibility and high rainfall reduce the average weekday's traffic speed upto 40 mph during congested hours. This speed profiles of different weather patterns can be combined with the traffic volume information to generate a city-scale variation of the emission and fuel consumption due to weather. Also, the vehicle trajectory data can be used in tandem to analyze weather impacts on the vehicle's start and stop behavior, which is difficult to capture only by using speed data.

5. Conclusion

Transportation sustainability measures such as emission, fuel consumption, delay cost, etc. are used as performance indicators of a transportation system. Weather is one of the factors that impacts these sustainability measures. Weather patterns such as high rainfall, low visibility could cause a conservative driving behavior, which leads to the traffic congestion, that in-turn increases the emission and fuel consumption. In this work, we have performed a data-driven analysis on the impacts of different weather patterns on the average hourly traffic speed in the city of Chicago. Through this analysis, we have shown that there is a (statistically) significant impact of different weather patterns on the average hourly traffic speed and it varies geographically across different land-use and road types. In this work, we have used only speed data (probe) and carried out the an aggregate-level analysis. We are extending this work by developing a neural network based predictive model to estimate average hourly traffic speed using weather and spatial variations (i.e. land-use and road types) as predictors. Further, this model will be used to generate various scenarios for a transportation simulation model, which will be further executed to analyze the detailed impact of weather patterns on traffic and fuel consumption.

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